Equivalent fractions

We already know how to simplify a fraction to lowest terms. We just pull out the common factors from the numerator and denominator, cancel those out, and what's left is the fraction simplified to lowest terms, resulting in an equivalent fraction. For example, given the fraction

$$\frac{30}{45}$$

we first find the prime factorizations of the numerator and denominator.

$$\frac{3 \cdot 5 \cdot 2}{3 \cdot 5 \cdot 3}$$

Then we cancel the 3 in the numerator with one of the 3's in the denominator, and cancel the 5 in the numerator with the 5 in the denominator, because $3/3$ and $5/5$ are equivalent to 1. The end result we're left with is $2/3$.

$$\frac{\cancel{3} \cdot \cancel{5} \cdot 2}{\cancel{3} \cdot \cancel{5} \cdot 3} = \frac{2}{3}$$

So we can say that the two fractions $30/45$ and $2/3$ are "equivalent", which just means that they're equal to each other.

$$\frac{30}{45} = \frac{2}{3}$$

In this lesson, we're turning this process around, and learning how to express something like $2/3$ in terms of 12ths in the denominator, instead of 3rds in the denominator.

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Example

Express $3/5$ as an equivalent fraction, but with 10 in the denominator instead of 5.

If we start with the fraction $3/5$, you could say that we have "3 out of 5 parts." Since we're being asked to express this as an equivalent fraction with 10 in the denominator, we're being asked the question "If we have 3 out of every 5 pieces, how many pieces would we have if there were 10 total pieces?"

Mathematically, here's how we set that up:

$$\frac{3}{5} = \frac{?}{10}$$

Now the question is, how did we get from 5 to 10 (in the denominator)? Well, we had to multiply 5 by 2 in order to get to 10. Which means that, in order to figure out what should go in place of the question mark, we need to multiply the numerator 3 by 2 as well, because we aren't allowed to change the overall value of the fraction:

$$\frac{3}{5} = \frac{3 \cdot 2}{5 \cdot 2} = \frac{6}{10}$$

This means that $3/5$ and $6/10$ are equivalent fractions.

Here's how we can double-check our answer. We'll reduce $6/10$ to lowest terms, and show that we get $3/5$:

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$$\frac{6}{10} = \frac{3 \cdot 2}{5 \cdot 2} = \frac{3 \cdot \cancel{2}}{5 \cdot \cancel{2}} = \frac{3}{5}$$

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